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Engineering Notes

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Orbit Design for Ground Surveillance Using Genetic Algorithms

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Introduction

THE orbits of remote sensing systems usually dictate the ground resolution, area coverage, and frequency of coverage parameters. Lower altitude orbits enable a spacecraft to provide higher resolution measurements, but orbital perturbations are nonnegligible due to atmospheric drag [1]. Electric propulsion systems can be used to compensate for atmospheric drag [2]. However, low altitude orbits lack wide coverage. Strategies to overcome the lack of coverage, by maneuvering the spacecraft using electric propulsion, were proposed in the literature [3]. In this case, the thrusters are not only used to compensate the perturbations, but are also used to continuously maneuver the spacecraft to achieve given coverage requirements over a given set of sites in a given time frame. One example is to visit a set of 20 sites within a time frame t_f of 50 days. However, this solution complicates the satellite system.

The motivation then is to find a lower cost solution for this kind of mission. This note proposes a new solution in which the spacecraft is placed in a natural orbit such that it visits all the sites within the time frame without maneuvering. An advantage is the short time for visiting all sites. Performing continuous thrust transfers to maneuver the satellite from one site to another often takes more time. The disadvantage is that some sites may not be visited accurately. A tradeoff between the accuracy and the revisit time for all the sites is needed when planning a mission.

To find this natural orbit, we use a genetic algorithm (GA) to perform a directed search among all possible orbits. Implementing a genetic algorithm does not guarantee an optimal orbit, however, it has been shown that in subsequent iterations, better solutions will be sampled at exponentially increasing rates [4]. This issue will be discussed in the section titled Genetic Algorithms.

GAs have been adopted in the literature to solve several orbital mechanics problems. We chose to solve the ground surveillance problem using genetic algorithms because this problem is characterized by many local minima. Conventional optimization methods (e.g., gradient methods) are not suitable for this kind of problem. GAs use random choice as a tool to guide a highly exploitative search in the design space [5]. Enumerated methods scan the whole domain and find the optimal solution. Enumerated methods can provide good solutions to the problem. However, the efficiency of these algorithms is very low compared with genetic algorithms [5].

The GA solution is used as a starting point to find a local minimum solution through traditional optimization methodologies. The final solution will be a local minimum in the neighborhood of the GA solution.

Two types of constraints are considered. The first mission searches for maximum resolution for each site for a given imaging sensor. The second mission tries to maximize the observation time.

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Problem Formulation

Assume we have n sites to be visited. Each site is defined by its geodetic longitude and latitude λ_k and ϕ_k, respectively, where $k = 1, \ldots, n$. The difference between geodetic and geocentric latitudes is usually very small [6], and is neglected in this analysis.

The position vector for the kth site in the Earth-centered inertial (ECI) frame is

$$\boldsymbol{r}_k^I = R_{\mathrm{E}} \begin{Bmatrix} \cos\phi_k \cos(\lambda_k + \omega_E t) \\ \cos\phi_k \sin(\lambda_k + \omega_E t) \\ \sin\phi_k \end{Bmatrix} \tag{1}$$

The only variable in the position vector of the kth site is the time t at which this site will be visited.

The satellite position $\boldsymbol{r}^O$ can be expressed in the perifocal coordinate system (PQW) as [7]

$$\boldsymbol{r}^O = \frac{p}{1 + e\cos\varphi} \begin{Bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{Bmatrix} \tag{2}$$

This vector is transformed to the ECI coordinate system through the transformation matrix [7] $R^{I/O}$ ($C \equiv \cos$ and $S \equiv \sin$)

$$R^{I/O} = \begin{bmatrix} C_\omega C_\Omega - C_i S_\omega S_\Omega & -S_\omega C_\Omega - C_i S_\Omega C_\omega & S_i S_\Omega \\ C_\omega S_\Omega + C_i S_\omega C_\Omega & -S_\omega S_\Omega + C_i C_\Omega C_\omega & -S_i C_\Omega \\ S_\omega S_i & C_\omega S_i & C_i \end{bmatrix} \tag{3}$$

by

$$\boldsymbol{r}^I = R^{I/O} \boldsymbol{r}^O \tag{4}$$

The time in Eq. (1) is coupled with the true anomaly in Eq. (2) through the Kepler equation.

Consider a satellite with an observing instrument (radar, camera, etc.) with an aperture of ϑ_{FOV}. Two cases are considered: one is to maximize the resolution, and one is to maximize the observation time.

For the best resolution case, a candidate optimality criterion is to minimize a weighted sum of squares of the distances between each site and the satellite at the nearest ground track point. The penalty function

$$L_R = \sum_k \alpha_k (\boldsymbol{r}_k^I - \boldsymbol{r}^I)^T (\boldsymbol{r}_k^I - \boldsymbol{r}^I) \tag{5}$$

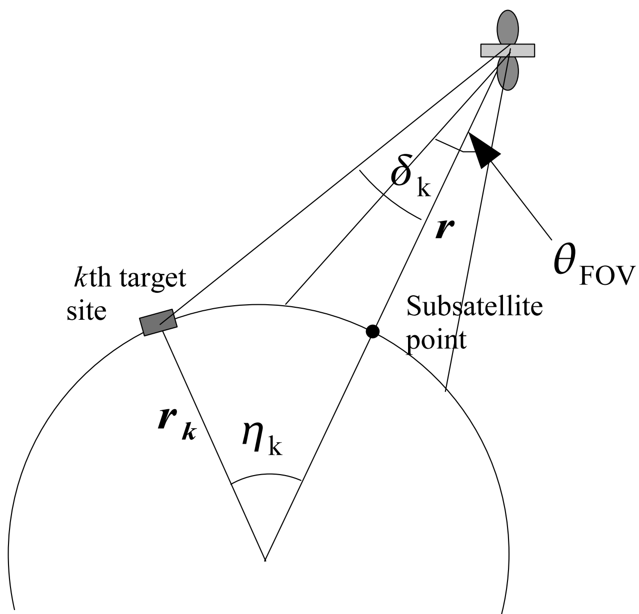


Fig. 1 The angle η_k as defined for the kth site.

will drive a solution orbit to pass as near as possible to each site and also have the best achievable resolution because the resolution is proportional to

$$\| (\boldsymbol{r}_k^I - \boldsymbol{r}^I) \|$$

For the observation time, the penalty function is

$$L_T = \sum_k \alpha_k H\left(\frac{1}{2}\vartheta_{\mathrm{FOV}} - \delta_k\right)\left(t_f - \int_0^{t_f} \cos\eta_k \,\mathrm{d}t\right) \qquad (6)$$

where η_k is the angle between $\boldsymbol{r}$ and $\boldsymbol{r}_k$ (see Fig. 1), $H(x)$ is the Heaviside unit step function [$H(x) = 0$ if $x < 0$, $H(x) = 1$ if $x > 0$], and δ_k is the nadir angle, measured at the satellite from the nadir to the site.

The objective is to minimize the penalty function L, which is a function of a state vector whose elements are the orbital parameters a, e, i, ω, and Ω, and all the visiting times t_k. A visiting time t_k is the time of closest approach to the site k. The minimizing state vector dictates the solution orbit.

If the solution orbit is not satisfactory, the set of target sites can be split into two subsets, and each subset is solved separately. The solution consists of two orbits, and the satellite has to be maneuvered between the two orbits to complete the mission. Algorithms for splitting the sites and maneuvering the satellite are available in the literature [8].

Genetic Algorithms

Genetic algorithm techniques are used to search for the minimum of the penalty function. The selection operator implemented in this note is the roulette wheel selection. Using this selection operator, the probability of selecting an individual is proportional to its fitness [4]. Any two selected individuals may, or may not, undergo the crossover operation based on the crossover probability P_c. The single point crossover operator is implemented in this note. Mutation is the switching of a bit in a member string from one to zero or vice versa. Each bit in each string is mutated with a probability of mutation P_m [4].

Each generation represents one iteration in the search process. The fundamental theory of GAs proves that in subsequent iterations, the most fit members will be sampled at exponentially increasing rates. This process leads to the evolution of populations of individuals that are more fit [5]. So, although using a GA does not guarantee an optimal solution, it samples the most fit members and finds the best member among them.

The population size depends mainly on the size of the problem. Computational cost increases with population. Exploring the design space can be achieved by increasing the crossover probability for a given population size [4]. The population size is selected along with the crossover probability such that the algorithm is computationally efficient and robust. For our problem, a population size of 100–200 members is used. The crossover probability ranges generally from 0.6 to 1 [4], and in this application it depends on the number of sites. A parametric study is performed for different values of the number of sites. For instance, for a case of 5 sites with a population size of 200 and a mission duration of 3 days, the crossover probability is 0.95. The algorithm converges after about 100 generations. Different recommendations for the mutation probability are available in literature [4]. After a parametric analysis, the mutation probability is selected to be 0.001.

For the maximum resolution case, the fitness of a member is evaluated as follows. For each site, the mean anomaly is calculated as a function of the visiting time for this site. The time is a design variable and is generated by the GA operations. Mean anomaly and eccentricity are used to find the true anomaly by solving Kepler's equation. True anomaly is used to calculate the satellite position, Eq. (2). The satellite position is compared with the site position to calculate the penalty for this site. These calculations are repeated on all the sites and added to calculate the penalty function L_R for each member. The fitness of a member is

$$\text{fitness} = -L_R = -\sum_k \alpha_k (\boldsymbol{r}_k^I - \boldsymbol{r}^I)^T (\boldsymbol{r}_k^I - \boldsymbol{r}^I) \qquad (7)$$

Results

A simple case of having only two sites is first examined. The mission duration time is three days in this example. Repeat ground track orbits can be designed by imposing constraints on the semimajor axis value. The penalty function used for the maximum

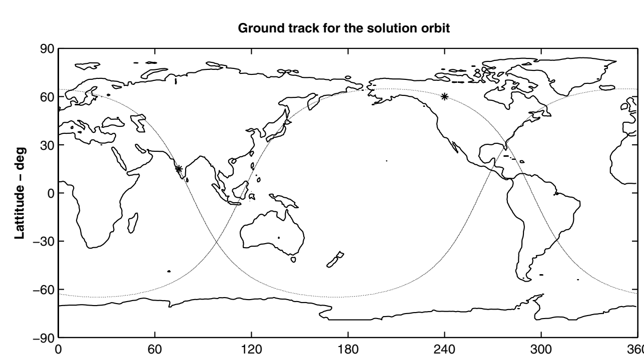


Fig. 2 Optimal orbit ($n = 2$ sites , $t_f = 3$ days).

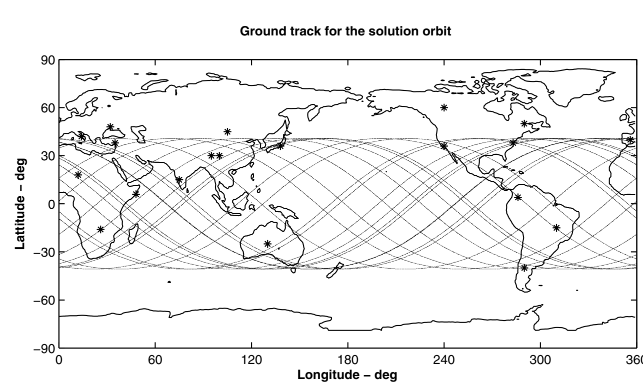
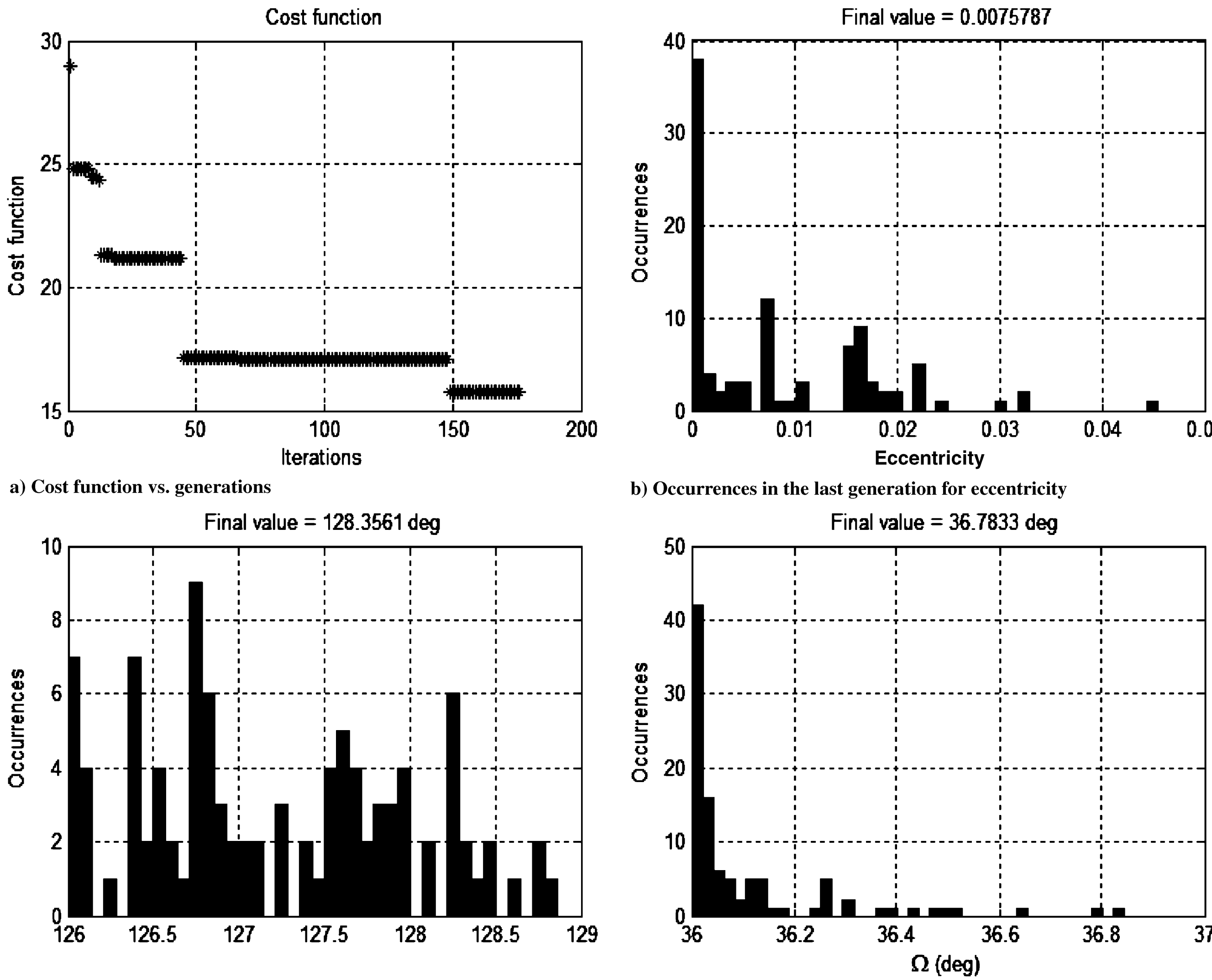


Fig. 3 High resolution mission ($n = 20$ sites, $t_f = 3$ days).



a) Cost function vs. generations

b) Occurrences in the last generation for eccentricity

c) Occurrences in the last generation for inclination

d) Occurrences in the last generation for RAAN

Fig. 4 Histogram (min L_R, $n = 20$ sites).

resolution case L_R is a nondimensional measure for how far the solution is from an ideal solution. The expected optimal solution is an orbit whose ground track is as close as possible to the sites. The semimajor axis a is not included in the design variables. Because resolution is proportional to slant range, the algorithm would find orbits too close to the ground if a were a design variable.

Figure 2 shows the ground track of the optimal orbit for this case. The orbit parameters are $a = 7753.5$ km, $e = 0.7$, $i = 40$ deg, $\omega = 330$ deg, and $\Omega = 362$ deg. The penalty function, at convergence, reached the value of $L_R = 2.58 \times 10^{-9}$. From Fig. 2, the orbit passes nearly directly over each site. The penalty function is almost zero.

Figures 3 and 4 show the results for a case study of 20 sites with a mission duration of 3 days. The solution for this case is achieved after 150 generations. The highest resolution is also required in this mission. The solution orbit semimajor axis is set to 6892.8 km. The resulting orbit parameters are: $e = 0.045$, $i = 40.8$ deg, $\omega = 145.7$ deg, and $\Omega = 86$ deg. The visiting times at which each site is visited are calculated. We present these times by indicating the satellite revolution in which the site is visited.

The solution resulting from the GA has a penalty function of $L_R = 16$. Gauss–Newton optimization algorithm is used to refine the solution. The solution provided by the GA is used as a starting point. The final value for the penalty function after refinement is $L_R = 3.23$. The solution resulting from the GA is shown in Fig. 4. The occurrences of the orbital elements in the last population as well as the history of convergence are also shown. From the histogram, all members in the last population have eccentricities less than 0.05, all inclinations are between 126 and 129 deg., and all right ascension of ascending nodes (RAANs) are between 36 and 37 deg. Recall that the members in the initial population have random values for these parameters. It is clear that all the initial random members converged to a small region in the design space. The orbit revolutions, in which each target site is visited, is listed in Table 1

Figures 5 and 6 show the results for the second optimality definition L_T, where the objective is to maximize the observation time. The mission duration is also three days. Figure 5 shows the solution for the case of 20 sites. The resulting orbit parameters are:

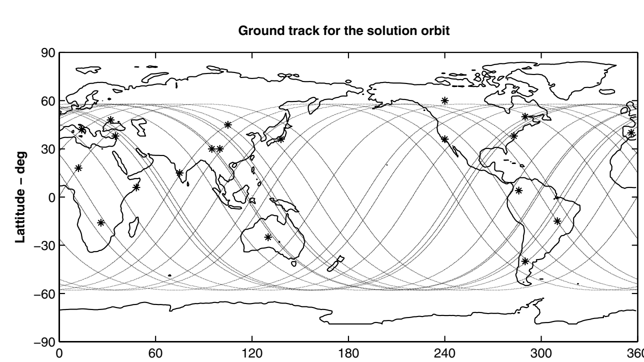
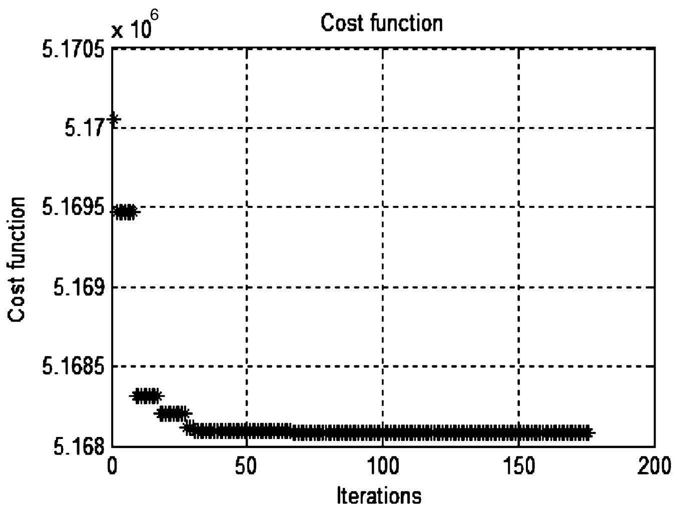
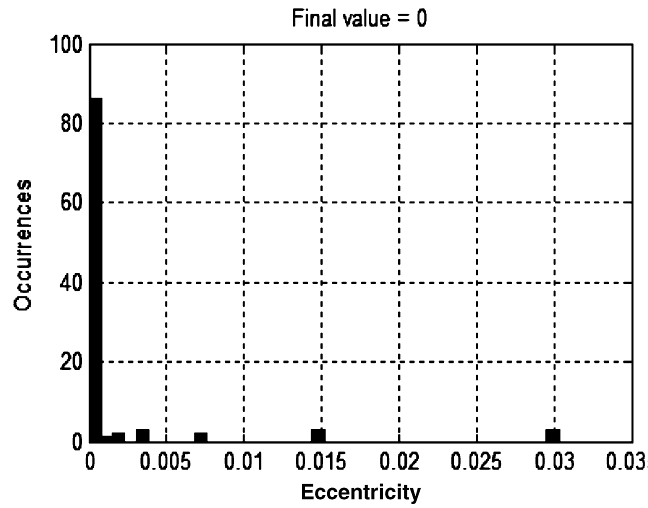


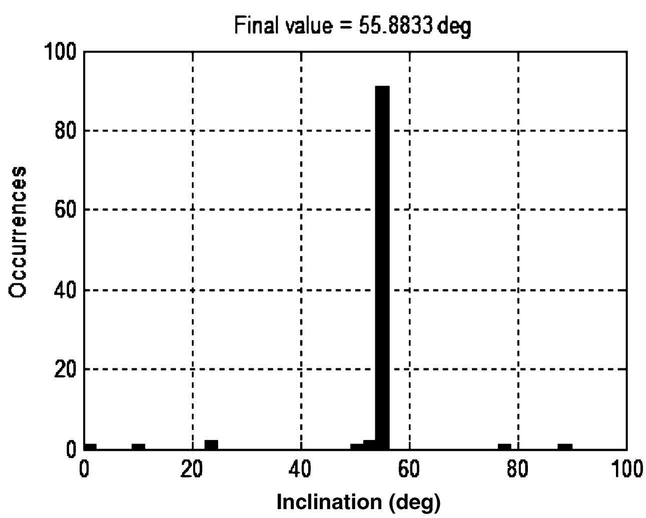
Fig. 5 Optimal orbit (min L_T and $n = 20$ sites).



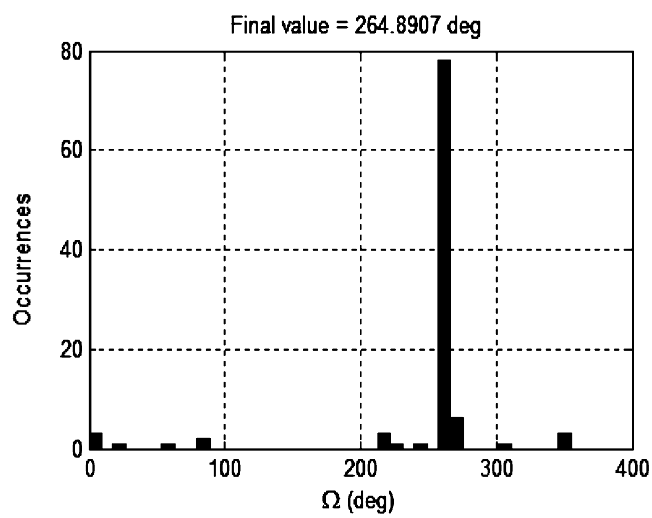
a) Cost function vs. generations



b) Occurrences in the last generation for eccentricity



c) Occurrences in the last generation for inclination



d) Occurrences in the last generation for RAAN

Fig. 6 Histogram (min L_T, $n = 20$ sites).

$a = 6882.5$ km, $e = 0.0$, $i = 58$ deg, $\omega = 339$ deg, and $\Omega = 238.8$ deg. The visiting orbit revolutions are shown in Table 2. Figure 6 shows the results of the GA iterations.

Disturbances, like the aerodynamic drag and solar radiation pressure, cause the orbit to decay. The orbit must be maintained by controlling the effect of disturbances. Assume electric propulsion is used for the orbit maintenance in the case of 20 sites with a duration time of 3 days. Algorithms for orbit maintenance and estimates for the required thrust and fuel mass using electric propulsion are available in the literature [2]. If the thruster specific impulse is 6000 s and the spacecraft mass is 200 kg, the amount of fuel needed for orbit maintenance in this case is about 0.25 g/day. The maximum thrust needed is about 0.3 mN. The thruster will not operate continuously but rather with a duty cycle that ranges from 47 to 85%. However, if using the electric propulsion to maneuver the spacecraft continuously between sites, estimates [3] show that, for the case of 20 sites with a duration time of 50 days, the amount of fuel needed is 5 g/day. The maximum thrust needed is about 1 mm/s^2, and the thruster is working continuously.

Conclusion

This note demonstrates the possibility of using genetic algorithms to search for natural orbits that achieve ground surveillance mission requirements. For a mission duration of three days, the genetic algorithm technique found the global optimal natural orbit in the case of any two sites. The solution to the case of three sites is near optimal with a small error. As the number of sites increases, the solution orbit does not visit each site.

Table 1 Observed orbits sequence (min L_R and $n = 20$)

Site	1	2	3	4	5	6	7	8	10	11	12	14	15	16	17	18	19	20
Orbit revolution	24	9	3	14	17	1	1	24	12	18	7	3	40	26	6	1	35	5

Table 2 Observed orbits sequence (min L_T and $n = 20$)

Site	1	2	3	4	5	6	7	8	10	11	12	14	15	16	17	18	19	20
Orbit revolution	11	1	21	31	3	4	42	14	35	13	27	15	37	33	28	13	15	38

References

[1] Chen, H. S., "*Space Remote Sensing Systems, An Introduction*," 16th ed., Academic Press, Orlando, FL, 1985, pp. 204–231.

[2] Abdelkhalik, O., "Remote Sensing Satellites Orbits Design and Control," M.S. Dissertation, Department of Aerospace Engineering, Cairo Univ., Giza, Egypt, 2001, pp. 54–60.

[3] Guelman, M., and Kogan, A., "Electric Propulsion for Remote Sensing from Low Orbits," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 2, 1999, pp. 313–321.

[4] Coly, D. A., "*An Introduction to Genetic Algorithms for Scientists and Engineers*," World Scientific Publishing, River Edge, NJ, 1999, pp. 17–58.

[5] Goldberg, D. E., "*Genetic Algorithms in Search, Optimization, and Machine Learning*," Addison–Wesley, Cambridge, MA, 1989, pp. 27–57.

[6] Vallado, D. A., "*Fundamentals of Astrodynamics and Applications*," 2nd ed., Microcosm, El Segundo, CA, and Kluwer Academic, Norwell, MA, 2001, pp. 135–147.

[7] Sidi, M. J., "Spacecraft Dynamics and Control," *A Practical Engineering Approach*, Cambridge Univ. Press, New York, 1997, pp. 20–28.

[8] Abdelkhalik, O., "Orbit Design and Estimation for Surveillance Missions Using Genetic Algorithms," Ph.D. Dissertation, Department of Aerospace Engineering, Texas A&M Univ., College Station, TX, 2005, pp. 38–60.